LOCAL GRAYSCALE GRANULOMETRIES BASED ON OPENING TREES

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Abstract.
Granulometries are morphological image analysis tools that are particularly useful for estimating object sizes in binary and grayscale images, or for characterizing textures based on their pattern spectra (i.e., granulometric curves). Though granulometric information is typically extracted globally for an image or a collection of images, local granulometries can also be useful for such applications as segmentation of texture images. However, computing local granulometries from a grayscale image by means of traditional sequences of openings and closings is either prohibitively slow, or produces results that are too coarse to be really useful. In the present paper, using the concept of *opening trees* proposed in [14], new local grayscale granulometry algorithms are introduced, that are both accurate and efficient. These algorithms can be used for any granulometry based on openings or closings with line segments or combinations of line segments. Among others, these local granulometries can be used to compute *size transforms* directly from grayscale images, a grayscale extension of the concept of an *opening function.* Other applications include adaptive openings and closings, as well as granulometric texture segmentation.

Key words: Algorithms, Local Grayscale Granulometries, Opening Trees, Mathematical Morphology, Pattern Spectrum, Size Transforms, Texture Segmentation.

1. Introduction, Motivations

The concept of *granulometries,* introduced in the late sixties by G. Matheron [8, 9], provides a consistent framework for analyzing object and structure sizes in images. A granulometry can simply be defined as a *decreasing* family of openings (See [11]) , $= (\gamma_n)_{n\geq 0}$:

$$\forall n \geq 0, m \geq 0, \quad n \geq m \Longrightarrow \gamma_n \leq \gamma_m. \tag{1}$$

Though this definition was originally meant in the context of binary image processing, it directly extends to grayscale. Moreover, granulometries "by closings" can also be defined as families of increasing closings.

Performing the granulometric analysis of an image I with , is equivalent to mapping each opening size n with a measure $m(\gamma_n(I))$ of the opened image $\gamma_n(I)$. This measure is typically chosen to be the *area* (number of ON pixels) in the binary case, and the *volume* (sum of all pixel values) in the grayscale case. The *granulometric curve,* or *pattern spectrum* [7] of I with respect to , , denoted $PS_\gamma(I)$ is then defined as the following mapping:

$$\forall n > 0, PS_\gamma(I)(n) = m(\gamma_n(I)) - m(\gamma_{n-1}(I)), \tag{2}$$

The pattern spectrum $PS_\gamma(I)$ maps each size n to some measure of the bright image structures with this size. By duality, the concept of pattern spectra extends to granulometry by closings, and is used to characterize size of dark image structures.

Granulometries are therefore primarily useful to extract *global* size information from images. For example, in [15, 14], granulometries based on openings with squares were used to directly extract dominant bean diameter from binary images of coffee beans; no segmentation (separation of touching beans) was required whatsoever. Similarly, Fig. 1 illustrates how granulometries can be used to robustly estimate structure size in grayscale images without any prior segmentation[1]. In other applications, granulometric curves are considered as feature vectors that characterize image texture and are used for classification. Though the discriminating power of these curves has theoretical limitations [10], experiments with automatic plankton identification from towed video microscopy images [2, 12] proved that granulometries can provide extremely reliable set of features (see Fig. 2).

The ability of granulometries to characterize textures makes it natural to think of using them *locally* for texture image segmentation [1, 3, 5]. Unfortunately, without special-purpose hardware, image segmentation using local granulometries can be an extremely slow process. Even the relatively efficient technique described in [5] requires a significant amount of computing power, as well as large amounts of memory.

Based on the concept of *opening trees* proposed in [14], new efficient local grayscale granulometry algorithms are introduced in the present paper. Section 2 first provides some reminders on opening trees and derived algorithms. In Section 3, these techniques are shown to be naturally suited to the extraction of local granulometries. Applications include the efficient extraction of grayscale *size transforms* [16], in which each image pixel is mapped to the size of the dominant bright (resp. dark) structure it is part of—a grayscale extension of the concept of an *opening transform* [6, 15]. The robustness of the proposed algorithms generally increases when the local granulometries are computed over a moving window. Efficient algorithms are also proposed for this case. All the techniques described work for granulometries with linear openings/closings and combinations thereof, some of which approximating the isotropic case.

2. Background on Opening Trees

Fast algorithms were proposed in [14] for linear grayscale granulometries and grayscale granulometries using openings (resp. closings) with combinations of linear structuring elements. One of the key concepts introduced there is that of an *opening tree:* given an image cross-section in any orientation θ, an opening tree T_θ can be used to compactly represent the successive values of each pixel of this cross-section when performing linear openings of increasing size in orientation θ. The opening tree therefore captures the entire granulometric information for this cross-section, as illustrated in Fig. 3.

Opening trees can be efficiently extracted from a gray image I in any orientation. They are especially useful for computing granulometries with maxima of linear

[1] Images gracefully provided by DMS, CSIRO, Australia.

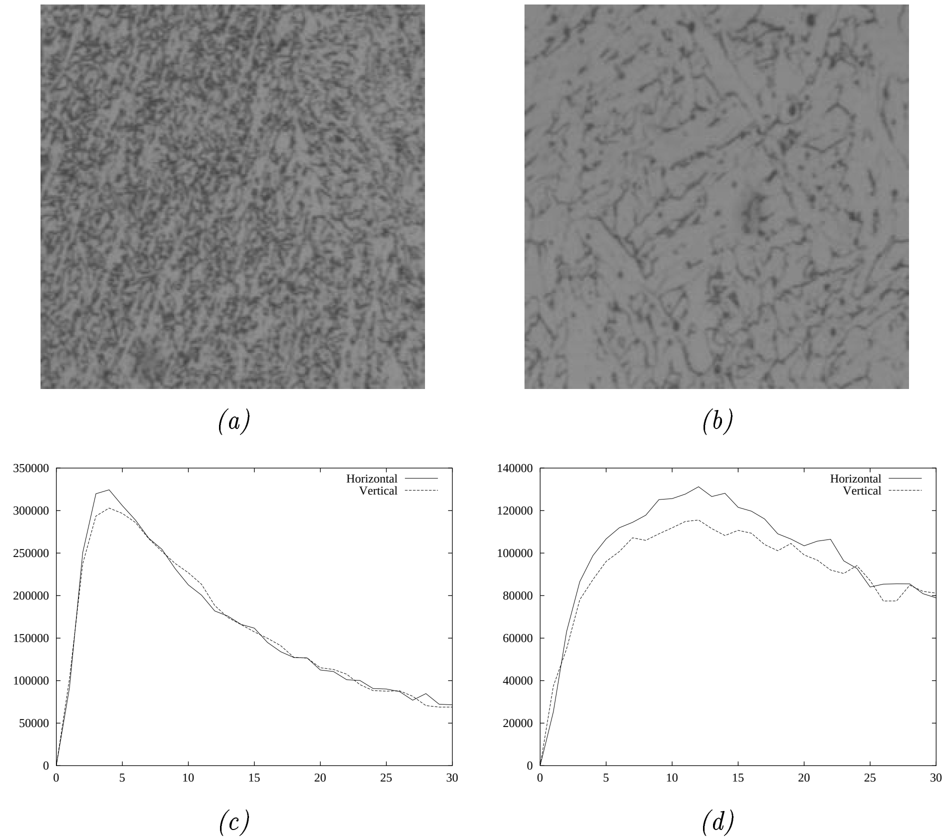
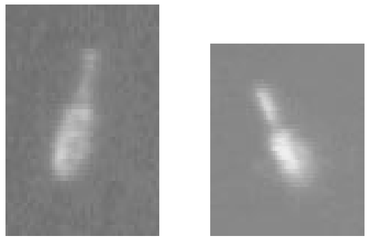
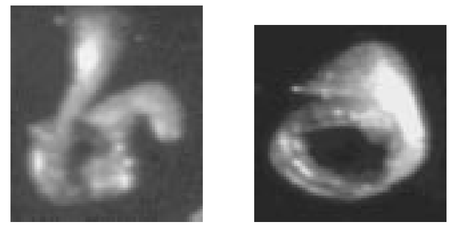


Fig. 1. Use of linear grayscale granulometries to estimate dominant width of white patterns in X-ray images of welds: curves in (c) clearly indicate that typical width of the white patterns in (a) is 4 pixels. Similarly, pattern spectra (d) show that typical pattern width in image (b) is 12 pixels. Vertical and horizontal granulometries provide the same answer in each case. On these 512×512 images, extraction of each curve takes about 0.2s on a Sun Sparc Station 10, using the algorithms introduced in [14].

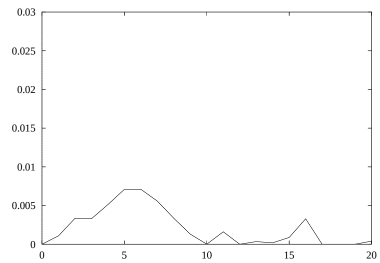
openings in two orientations θ_1 and θ_2: each pixel p of I is mapped to a leaf of opening tree T_{θ_1} and to a leaf of opening tree T_{θ_2}. By following these two trees simultaneously down to their root, the successive values of p through maxima of linear openings in directions θ_1 and θ_2 are efficiently derived. The corresponding granulometric curve of I follows immediately. This technique extends to a combination of any number of linear openings (resp. linear closings). Furthermore, *pseudo-granulometries* by minima of linear openings (resp. maxima of linear closings) can also be derived. While minima of linear openings are generally of little practical value, pseudo-granulometries based on such families of transforms tend to capture the same information as granulometries with convex elements such as squares, and have proved equally valuable for classification applications [12] (The curves of Fig. 2 are in fact pseudo-granulometries).



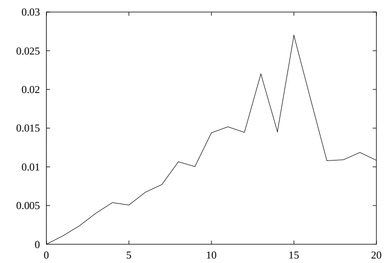
(a) Copepod oithona



(b) Pteropods



Typical curve for copepod oithona



Typical curve for pteropods

Fig. 2. Plankton classification using grayscale granulometries.

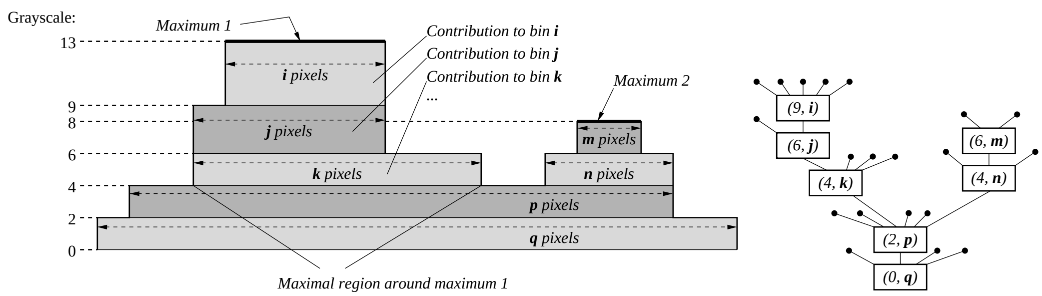


Fig. 3. Left: cross-section of a grayscale image exhibiting two maxima. Right: corresponding opening tree capturing all the linear granulometric information in the direction of the cross-section. The leaves of the tree correspond to image pixels. See [14] for more information.

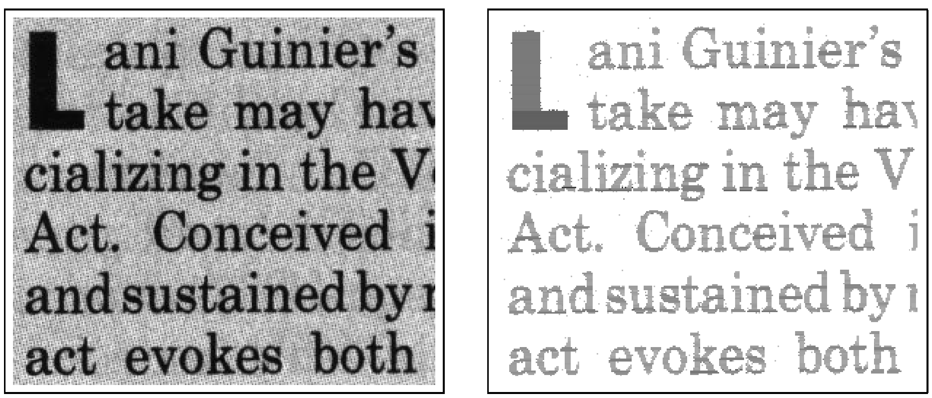
3. Local Granulometry Algorithms

The algorithms recalled in the previous section straightforwardly extend to local granulometries. Indeed, for each pixel p, following the associated opening tree(s) provides the successive values taken by this pixel through openings of increasing size: this is equivalent to the *local granulometry* information at p.

At each pixel location, the local granulometry information can be fed to a classifier (e.g., a neural network) in order to do texture segmentation [5]. The local pattern spectrum can also be used to derive a grayscale equivalent of the *opening transform.* In the binary case, the *opening transform* of an image I maps each pixel p to the size of the first opening such that the value of p in the opened image becomes 0. There are two simple ways to extend this concept to grayscale:

- Assign to each pixel the *sum* of its successive opening values.
- Assign to each pixel the opening size n that causes the *biggest drop in gray level* for this pixel, i.e., that maximizes $\gamma_n(I)(p) - \gamma_{n-1}(I)(p)$. (Note that with this definition, n is not necessarily unique.)

One can easily verify that, when applied to binary images, both transforms are equivalent to an *opening transform.* Both are also easily derived from the local granulometry information, i.e. the opening tree(s) at each pixel. The first transform was found to be only useful for very specific applications, whose description would be beyond the scope of the present paper. The second transform, already proposed in [16], is more generally useful. However, as pointed out by Vogt, its results are only meaningful when input image objects have "fairly clean boundaries". The concept of a *grayscale size transform* is illustrated in Fig. 4. Total computation time for this 290×252 image on a Sun Sparc Station 10 without particularly optimized code was of about 0.3s.



(a) original grayscale text image *(b) horizontal* size transform

Fig. 4. Example of *size transform* computed on dark image structures using "horizontal closing trees". Transform was restricted to dark pixels of original image. It maps each pixel to the local stroke width. Notice that stroke width is found to be largest inside the dropcap (large 'L' in top-left corner).

Instead of mapping each pixel p of grayscale image I to an opening (resp. closing) size n, another interesting transform, also proposed in [16], is derived by mapping p to its value in $\gamma_n(I)$, the opening of size n. The resulting transformed image can be described as an *adaptive opening:* at each pixel location in the image, the opening size is adapted to fit the size of the dominant structure. The corresponding *adaptive top hat transform* is particularly promising for automatic background normalization, when the size of image structures is either unknown (making it difficult to select the opening size for a classic *top hat transform*), or subject to large variations within the image. The opening-tree based implementation proposed here makes it possible to compute such adaptive openings/closings in typically under a second for most applications.

Unfortunately, experimentation showed that the above *size transforms* and adaptive openings/closings produce rather unstable results when computed on somewhat difficult or noisy input images. This can be attributed to the fact that granulometric information is usually not meaningful unless extracted over a large enough area—in fact, for most images, this information is meaningless at a pixel level. One workaround is to simply smooth or median-filter the *size transforms* or adaptive openings described above. A more robust technique consists of extracting the granulometric information for each pixel p over a reasonably large window centered at p.

More precisely, using the algorithms described in [14], the granulometric curve in a window $W(p)$ centered at pixel p of image I can be derived from the opening trees corresponding to the pixels in $W(p)$. Directly extracting granulometric information in each sub-window $W(p)$ for each pixel p would however be prohibitively expensive. To speed-up computation, we take advantage of the fact that for two neighboring pixels p and q, there is significant overlap between $W(p)$ and $W(q)$. Having computed the granulometric curve in $W(p)$, we can therefore derive the granulometric curve in $W(q)$ by:

1. subtracting the granulometric contribution of the pixels in $W(p) \setminus W(q)$, (where '$\setminus$' denotes set difference),
2. adding the granulometric contribution of the pixels in $W(q) \setminus W(p)$

These two operations are easily achieved by using the opening trees corresponding to pixels in $W(p) \setminus W(q)$ and in $W(q) \setminus W(p)$.

Such a scheme can be optimally implemented by scanning the image "like the ox plows the field", i.e., by scanning the first line from left to right, the second line from right to left, etc, until the last image line is reached. This is illustrated in Fig. 5. Note that this kind of technique can also be used to efficiently implement dilations/erosions by arbitrary structuring elements using moving histograms, and has even proved useful for binary dilations and erosions [13]. Computationally, a local granulometry operation such as a *size transform* computed over an $n \times n$ window is only be about $2n$ times slower than the "local" *size transform* described earlier in this section, which remains competitive. Experimentation showed that for most practical purposes, a window size of 10×10 to 40×40 was adequate.

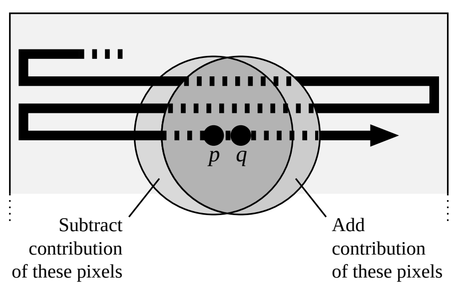
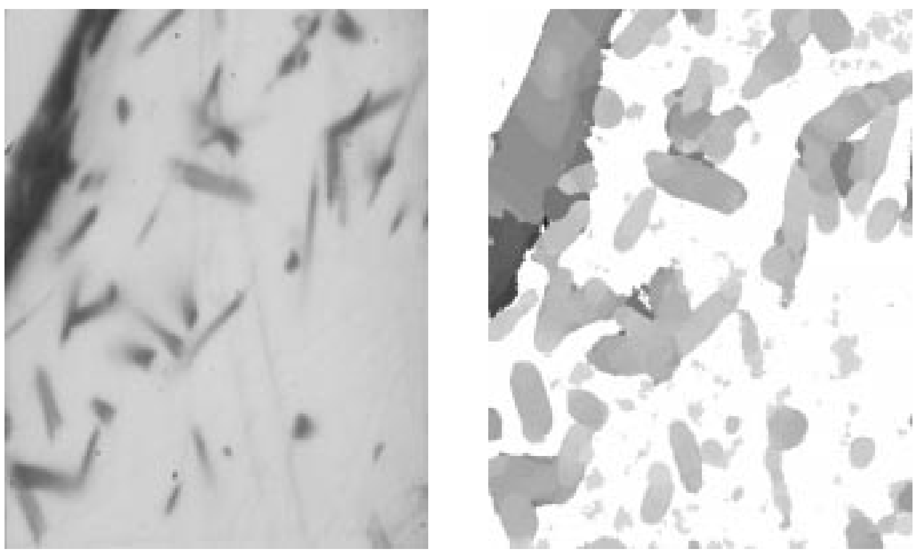


Fig. 5. Type of image scanning used for the efficient computation of local granulometries over moving windows (window is a disk here).

An example of a "regional" *size transform* is shown in Fig.6. Fig. 6a is an image of fission tracks in apatite. Such tracks form the basis of well-known techniques for dating rocks [4], and track length even provides thermal history information. The result of a *regional size transform* is shown in Fig. 6b. To produce this result, a disk-shape window of radius 15 was used. Also, since the intent was to characterize the size of dark structures, closing trees were used instead of opening trees. Notice that because of the circular window used by the algorithm, the dark structures appear dilated in the resulting transform.

In fact, for this example, "pseudo-granulometries" based on maxima of linear closings were used. Just like minima of linear openings are not openings, maxima of linear closings are not closings. However, when computed over large enough windows, these pseudo-granulometries tend capture size information that is very similar to what can be extracted using actual 2-D granulometries [12]. *Regional size transforms* based on these pseudo openings and closings are therefore useful operations. On the other hand, adaptive openings based on minima of linear openings and adaptive closings based on maxima of linear closings should be avoided.



a. Fission tracks image   *b.* Regional size transform

Fig. 6. Example of *size transform* of black structures, computed over a moving disk of radius 15. Dark values in (b) correspond to large structures.

4. Conclusions

A comprehensive set of efficient local grayscale granulometry algorithms was proposed. Based on the notion of opening trees, these techniques can be used to compute any transform such that the value of a pixel in the transformed image may be derived from local granulometric information at this pixel. Local granulometric information can be computed either on a pixel by pixel basis, or integrated over moving windows, for added robustness. Among the most useful derived operations are *size transforms* of bright or dark structures, which are a grayscale extension of the classic *opening transforms* and *closing transforms.*

The proposed techniques are valid for granulometries based on linear openings/closings and combinations thereof. True 2D granulometries are not handled yet. However, this was found not to be a major drawback in practice: in many cases, "pseudo-granulometries" with minima of linear openings or maxima of linear closings provide an adequate substitute to 2D granulometries. Overall, the set of methods described in this paper make local granulometries a computationally competitive alternative for texture segmentation problems.

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