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Valuation of image extrema using alternating filters by reconstruction

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ABSTRACT

Image extrema are often used for locating the structures present in an image. Their extraction and their selection is a classic image segmentation preprocessing problem. One of the most powerful morphological tools for selecting significant extrema in a grayscale is to use their *dynamics*. However, a drawback of this technique is that minima and maxima (the dark and light structures they point out) are processed independently.

We show in the paper that using the dynamics comes down to measuring the persistence of image minima (resp. maxima) when processing the image via a increasing (resp. decreasing) family of contrast filters. This principle can be generalized to any increasing family of morphological *filters by reconstruction* and leads to a general method for valuating image minima with respect to any criterion: size, shape, contrast... This proposition is still true for families of alternating filters whose main characteristic is to have a self dual behavior. In this paper we concentrate on this point. A symmetrical equivalent for the dynamics is defined and an efficient technique of computation is proposed. One of its key concept is a merging tree of extrema. The usefulness of this notion in image segmentation applications is also illustrated.

Keywords: Image analysis, Mathematical Morphology, Extrema, Dynamics, Aternating Filters, Grayscale Reconstruction, Segmentation.

1 INTRODUCTION

In many image analysis problems one is interested in locating the significant "structures" or "regions" present in an image. In morphology, this is typically done by first extracting "markers" of these significant structures, and then using the watershed transformation[1,2] to extract the contours of these structures as accurately as possible.

The purpose of the watershed is to assign an influence zone to markers pointing at the significant regions to be segmented in the image. The meaning of the expression *significant regions* depends on the context and is generally defined within the charateristics of the regions: their size, their contrast, their shape...

Extrema in grayscale images are often used to crudely extract the structures present in an image: if the image is considered as a topographic relief where gray levels correspond to altitude information, the dark and light structures of the image correspond to the valleys and the domes of the relief. The plateaus located at the top of the domes and the bottom of the valleys respectively correspond to *regional maxima* and *minima.*

Definition 1.1 (Regional minimum). *Let f be a mapping from a compact $C \subset \mathbb{R}^2$ onto $\mathbb{R}$. A regional minimum M of f is a connected set of a given altitude h, such that starting from M it is not possible to reach another point of lower altitude without having to climb.*

Unfortunately, for complex images, this information is not directely useful because of the great number of extrema present in the image: most of them correspond to insignificant structures or noise and have to be eliminated.This is the typical issue one faces when using the watershed transformation in an unconstrained way[1,2] (see figure 1).

Figure 1: Over-Segmentation produced by the watershed transform computed with all the image extrema

Rather than preventing the over-segmentation problem, the selection of the image extrema allows to control the segmentation produced : the characteristics and the number of the extracted regions. One of the most powerful tool provided by mathematical morphology in this domain is the dynamics.[4,5]

Definition 1.2 (Dynamics of a regional minimum). *Let f be a mapping from $C \subset \mathbb{R}^2$ onto $\mathbb{R}$ and M be a regional minimum of f. The dynamics of M is the minimal height one has to climb starting from M to reach another minimum of lower altitude.*

The dynamics of a regional maximum M is defined as well by considering the path of minimal altitude linking M to another maximum of higher altitude.

This transformation assigns to each image minimum or maximum a value that characterizes the contrast of the structure it marks. The selection of the most contrasted regions can then be made by a simple thresholding. One of its major interest is that the threshold level can be derived from the dynamics distribution : for example, it allows to extract markers of the n most contrasted objects or regions of the image (see figure 2).

An important aspect of dynamics is that it does not consider the size or the shape information (the small details of the eyes are selected in figure 2 for example). This positive aspect may become a negative one when this information has to be considered. For this reason, dynamics is generally used in parallel with spatial filters such that small and contrasted structures corresponding to noise are eliminated. We recently introduced a general method for valuating image extrema with respect to any criteria (not only contrast criterion).[12] A brief reminder of this approach is provided in section 2.2.

Another important aspect of dynamics is that image minima and maxima (i.e. dark and light structures) are processed independently. In the case of complex images, when dark and light structures are nested, this remark is important. In the second part of this paper, a symmetrical equivalent of dynamics is proposed. The usefulness of this notion in segmentation applications is illustrated in the last part of this paper.

Original image | Extrema of highest dynamics | Gradient image | Mosaic image (90 regions)

Figure 2: Segmentation of the most contrasted regions using dynamics

2 DYNAMICS AND MORPHOLOGICAL FILTERS BY RECONSTRUCTION

2.1 Foundations of dynamics

Throughout the paper, f will be a mapping from a compact set C of $\mathbb{R}^2$ onto $\mathbb{R}$. Let $T_h(f)$ be the threshold of f at value h:

$$T_h(f) = \{x \in C \mid f(x) \geq h\} \tag{1}$$

$\mathcal{M}in(f)$ ($\mathcal{M}ax(f)$) will denote the set of all regional minima (maxima) of f. A regional minimum M of f is a connected component of $T_h(f)$ such that :

$$\forall h' < h, T_{h'} \cap M = \emptyset \tag{2}$$

We have seen that the dynamics corresponds to a contrast measurement of the structures marked by the image minima or maxima. The standard algorithm used by mathematical morphology for removing the image dark structures whose contrast is lower than a given value h is a geodesic reconstruction by erosion of the translated image $(f + h)$ over f ($\epsilon^\infty(f, f+h)$)[9,15] (see figure 3):

$$\forall x \in C, \epsilon^\infty(f, f+h)(x) = inf\{t \geq f(x) \mid \gamma_x^c(T_t(f)) \cap T_{t+h}(f) \neq \emptyset\} \tag{3}$$

$\gamma_x^c(A)$ stands for the connected opening[9] which extracts the connected subset of A containing x.

$\epsilon^\infty(f, f+h)$ entirely removes the structures of contrast lower than h. The other structures are partially preserved: they are eroded (see figure 3).

Note that this operation can be simply used to extract the regional minima (i.e. the minima of dynamics greater or equal to 1) of a grayscale image[8]:

$$\mathcal{M}in(f) = \{x \in C \mid f(x) \neq \epsilon^\infty(f, f+1)\} \tag{4}$$

If $h > 1$, then extended minima called *h-minima* are extracted[13,7] and we have:

$$\forall M \in \mathcal{M}ax(\epsilon^\infty(f, f+h)), \exists N \in \mathcal{M}ax(f), N \subset M \tag{5}$$

In fact, the dynamics is closely linked to the family $(\epsilon^{\infty}(f, f+h))_{h\geq 0}$. Indeed, a regional minimum of f is still included in an extended regional minimum of $\epsilon^{\infty}(f, f+h)$ if and only if the structure it marks is of higher contrast than h, i.e. if and only if its dynamics is higher than h (see figure 3). So, the dynamics of a minimum M is the minimal "size" of the contrast filter to be computed so that M is totally eliminated:

$$\forall M \in \mathcal{M}in(f), dyn(M) = inf\{h \geq 0 \mid M \cap \mathcal{M}in(\epsilon^{\infty}(f, f+h) = \emptyset)\} \quad (6)$$

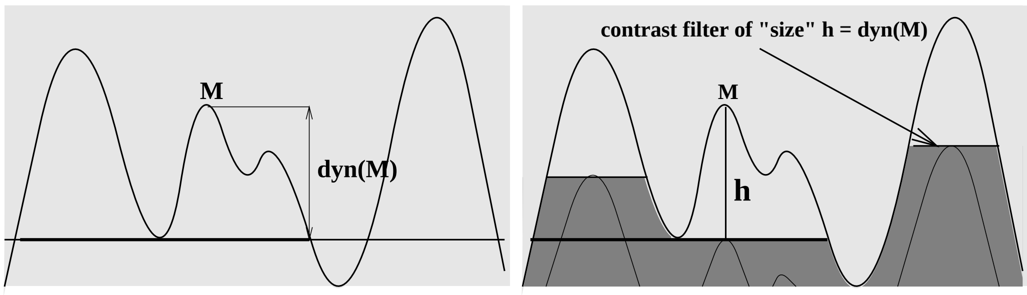

Figure 3: Link between dynamics and contrast filters by reconstruction

This relation introduces a new interpretation of dynamics: *dynamics corresponds to a measurement of the persistence of the image structures when applying increasing contrast filters.* An image minimum or maximum is then valuated with the persistence of the structure it marks.

This principle can be applied to other families of increasing morphological filters by reconstruction (not only contrast filters). This leads to a general method for valuating image extrema with respect to any criterion: size, shape, contrast...

2.2 General method for valuating image extrema

Let ψ_λ be an increasing family of morphological filters by reconstruction.[8,15] We focus on filters by reconstruction because they have very nice properties as regards the image extrema. Indeed, they correspond to *connected operators*[9,10,3] and just extend the extrema of the input image without creating new ones. For example: standard morphological openings or closings do not satisfy this property; they may break connected components, and therefore do not preserve the connectivity of image structures. On the contrary, openings and closings by reconstruction are connected operators (see figure 4).

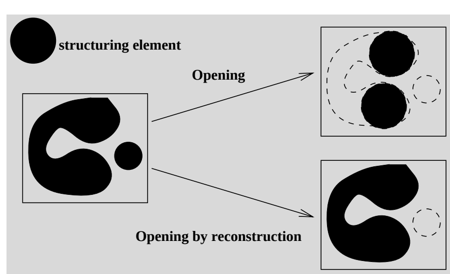

Figure 4: Difference between the opening and the opening by reconstruction

If ψ_λ acts on the image dark structures (i.e. ψ_λ is extensive : $\psi_\lambda(f) \geq f$), then it just extends the regional minima of the input image :

$$\forall M \in \mathcal{M}in(\epsilon^{\infty}(f, f+h)), \exists N \in \mathcal{M}in(f), M \supseteq N \tag{7}$$

Starting from the family $(\psi_\lambda)_{\lambda \geq 0}$ it is then possible to valuate each image minimum with the level λ for which it disappears[12]:

Definition 2.1 (Extinction values). *Let $\Psi = (\psi_\lambda)_{\lambda \geq 0}$ be an increasing family of extensive filters by reconstruction. The extinction value $\mathcal{E}_\Psi(M)$ of a regional minimum M of a mapping f associated with Φ is the maximal size of λ such that M is still a minimum of the filtered image $\psi_\lambda(f)$:*

$$\forall M \in \mathcal{M}in(f), \mathcal{E}_\Psi(M) = sup\{\lambda \geq 0 \mid \forall \mu \leq \lambda, M \cap \mathcal{M}in(\psi_\mu(f)) \neq \emptyset\} \tag{8}$$

The study of the image maxima requires families of anti-extensive filters.

Therefore, the dynamic of a minimum is related to the extinction value that would be assigned to this minimum when using a family of contrast filters by reconstruction in the previous definition. If the considered family is based on openings or closings by reconstruction, then the image extrema are valuated with respect to a size criterion. Area openings or closings[14] are particularly interesting: the associated *area extinction values* are independent of any shape criterion and then correspond to a spatial equivalent for dynamics[12] (see figure 5).

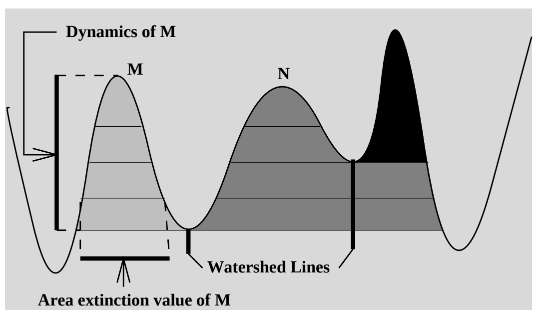

Figure 5: Comparison between dynamics and area extinction values

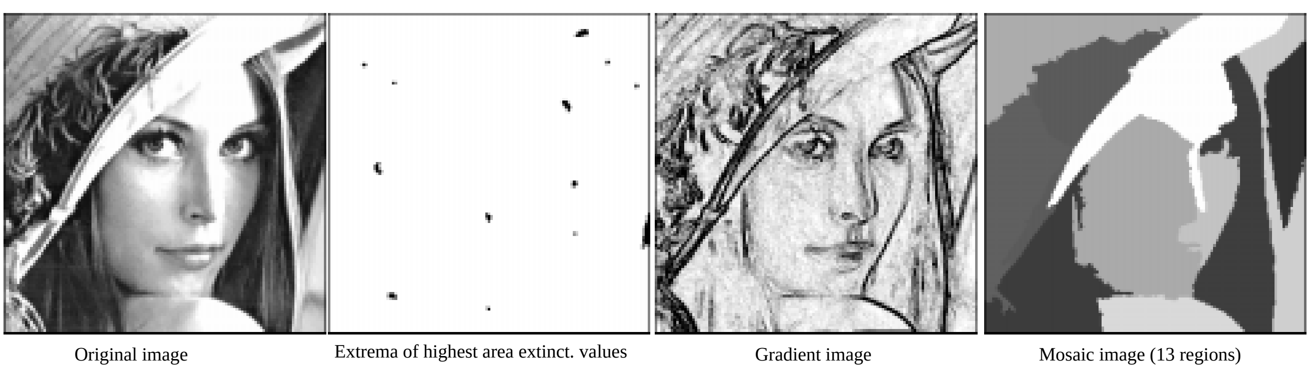

Figure 6: Segmentation of the most significant regions in terms of size

The figure 6 illustrates the segmentation obtained when the markers are defined as the 13 most significant extrema in terms of area (the area extinction values have been computed on the gradient image). This result compared to the figure 2 clearly shows the difference between dynamics and area extinction values : dynamics allows to extract the most contrasted regions regardless of their size, area extinction values allows to extract the largest regions regardless of their contrast.

3 VALUATION OF IMAGE EXTREMA USING ALTERNATING FILTERS BY RECONSTRUCTION

We have seen that the dynamics can be considered as an operator associated with a family of contrast filters by reconstruction. Those filters are non-symmetrical, i.e. they act either on the dark or on the light structures of the image. In this part, we propose a symmetrical equivalent for dynamics in which minima and maxima are simultaneously studied.

3.1 Definition

Let us consider an extensive transformation ψ_λ. ψ_λ only acts on the dark structures of the image. The dual transformation denoted $\overline{\psi}_\lambda$ defined by $\overline{\psi}_\lambda(f) = -\psi(-f)$ is anti-extensive and only acts on the light structures of the image. The pair of those dual transformations can be, size by size, alternately applied such that dark and light structures are both examined. The derived transformation is called *alternating sequential filters* (ASF)[9,11]:

$$\psi_\lambda^{ASF} = (\overline{\psi}_\lambda \circ \psi_\lambda) \circ (\overline{\psi}_{\lambda-1} \circ \psi_{\lambda-1}) \circ ... \circ (\overline{\psi}_1 \circ \psi_1) \qquad (9)$$

If ψ_λ is connected, then ψ_λ^{ASF} is connected too.[3,10] We have seen that filters by reconstruction only act on the image by extending its extrema. With increasing sizes of an ASF by reconstruction, one progressively gets images with more and more plateaus, originally corresponding to minima and maxima of the input image. This process produces a series of images of decreasing complexity (or level of detail): structures or regions in the image are progressively removed. The alternating sequential filters are not self-dual at all but they have a symmetrical behavior.

We propose to define symmetrical extinction values associated with families of increasing ASF by reconstruction as follows:

Definition 3.1 (Symmetrical extinction values). *Let ψ_λ^{ASF} be an alternating sequential filter. The symmetrical extinction value $\mathcal{E}^{sym}(M)$ of a regional extremum M of a mapping f is the maximal size of the ASF such that M is still an extremum of the filtered image:*

$$\forall M \in \mathcal{E}xtr(f), \quad \mathcal{E}^{sym}(M) = sup\{\lambda \geq 0 \mid \forall \mu \leq \lambda, M \cap \mathcal{E}xtr(\psi_\mu^{ASF}(f)) \neq \emptyset\} \qquad (10)$$

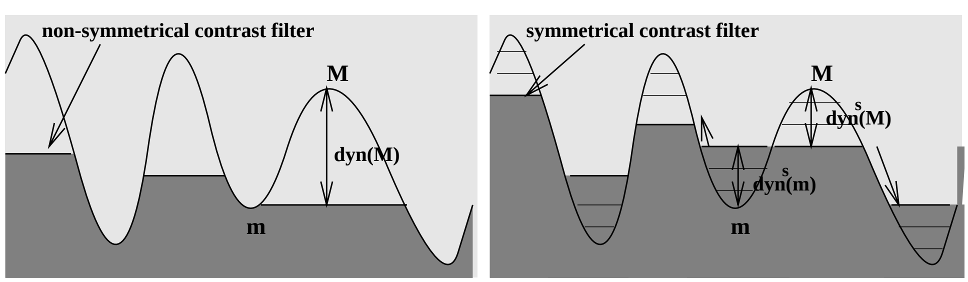

Figure 7: Difference between dynamics and symmetrical dynamics

If the filtering parameter is a contrast criterion (ASF defined by the pair $(\epsilon^\infty(f, f+h), \delta^\infty(f, f-h))$, then the associated extinction values define a symmetrical equivalent for dynamics we call *symmetrical dynamics* (noted dyn^{sym}) (see figure 7). This definition assumes the following inequality (see figure 7):

$$\forall M \in \mathcal{E}xtr(f), dyn^{sym}(M) \leq dyn(M) \qquad (11)$$

3.2 Efficient algorithm for computing symmetrical dynamics

The algorithm we propose for computing symmetrical dynamics is directly derived from the definition.

The first step consists in extracting and labeling the image extrema.[13] Then, to each extremum are progressively added its neighboring pixels, starting from those with the closest gray level. For this step, we use a hierarchical queue of pixels[13,6]: one pixel is entered in the queue with a priority level. If x is a neighboring pixel at altitude $f(x)$ of an extremum M whose initial altitude is denoted $f(M)$, x is then entered at the priority level : $\mid f(x) - f(M) \mid$.

The propagation scheme consists in extracting the pixels from the queue, level by level, starting with those of smallest priority level.

During the propagation scheme, several events may occur:

- An extended minimum (or maximum) has neighboring pixels of strictly lower (higher) altitude. In this case, the propagation of the region is stopped and its symmetrical dynamics is computed : it is exactly equal to the current priority level.
- When two labeled plateaus of same altitude meet, they merge: one of the two plateaus absorbs the other. Let M and M' be the two merging plateaus. The merging rules are the following: if M is not an extremum, then M is absorbed by M'; if M and M' correspond to two extrema, then the merging rule depends on the result of the union. For example, if M is a maximum and if the union between M and M' is still a maximum, then M absorbs M' (and the symmetrical dynamics of M' is computed) (see figure 8).

  This step is certainly the most delicate of our algorithm. Indeed, in order to know the type of the merging result (if it is a maximum, a minimum or a non extremum plateau), all the neighboring pixels of the two merging broadened plateaus (which were previously entered in the queue) have to be examined. Their altitude compared to the current altitude of the merging plateaus determines the type of the merging result. For this reason, we propose to associate with each initial extremum its own queue so that this step is facilitated. This solution presents the advantage not to be time consuming in itself.

  So, when two extended plateaus merge, their associated queues merge too. When the propagation of a plateau is stopped, then its associated queue is left pending.

The plateaus propagation is pursued until stability. During this process, an oriented merging tree of extrema is built: when two broadened extrema(M and M') meet, one absorbs the other (M absorbs M' for example) and one branch of the merging tree linking the nodes M and M' is created: $M' \longrightarrow M$ (see figure 8).

The information contained in this tree is very interesting. Indeed, we have seen that one property of the alternating sequential filters by reconstruction is to extend and merge the input image extrema. The tree allows to memorize when and how the extrema merge : one branch created at level h of the propagation scheme and linking two extrema M and M', expresses that M and M' are included in a same plateau of the filtered image resulting from the contrast ASF of size h.

Note that the same algorithm may be applied starting from the set of the labeled regional minima of the image. In this case, the standard dynamics is computed. The merging tree derived is then no longer symmetrical and just links the minima of the image (and not the maxima).

On a SPARC1 SUN station, the computation of the symmetrical dynamics on a 256×256 image takes from 10 seconds (for about 150 extrema) to more than 50 seconds (for more than 2000 extrema). In comparison, the computation of the dynamics (also based on the use of hierarchical queues) takes less than 8 seconds and does not depend on the number of minima treated. This significant difference comes from the merging step which occurs more frequently when the number of extrema is important and which requires many comparisons and is therefore time consuming.

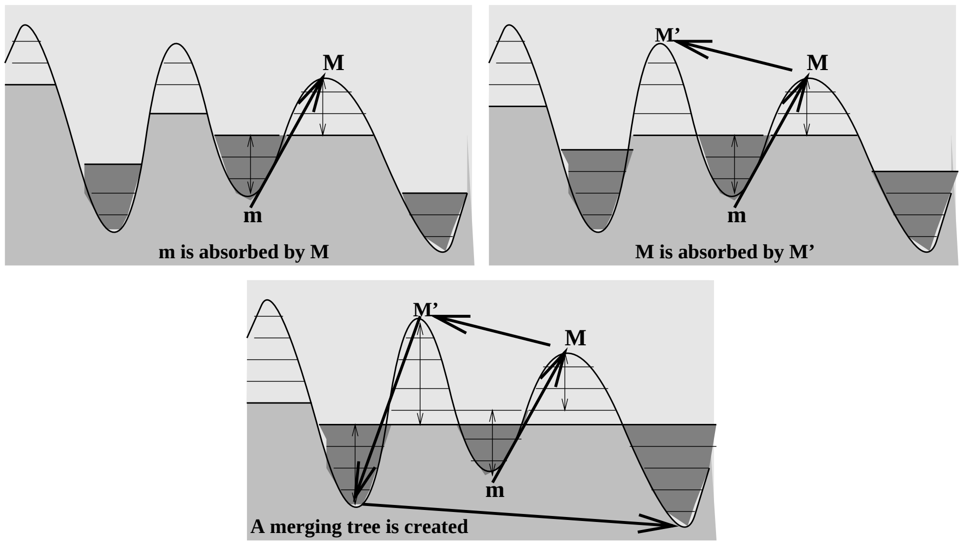

Figure 8: The symmetrical dynamics computation creates a merging tree of extrema

4 APPLICATION TO IMAGE SEGMENTATION

The figures 9 and 10 illustrate the comparison between the use of the dynamics and the use of the symmetrical dynamics. The markers are extracted by selecting the original image extrema of dynamics or symmetrical dynamics (computed on the original image) higher than 30. Note that the dynamics can be computed on the gradient image, but the signification of the associated valuation is not the same. Then the final segmentation is obtained by computing the watershed transform (the gradient image used is the same in the both cases). As we can seen in this example, the difference between dynamics and symmetrical dynamics is not significant.

In fact, this result was predictable. Indeed, there is almost no difference between an alternating sequential filter $((\overline{\psi}_\lambda \circ \psi_\lambda)...(\overline{\psi}_1 \circ \psi_1))$ and an alternating filter $(\overline{\psi}_\lambda \circ \psi_\lambda)$ and those filters are at the basis of the concepts of dynamics and of symmetrical dynamics.

Let us now compare this result to the segmentation one obtains when the markers are the extrema of the contrast ASF of same size 30 (see figure 11). The number of regions is the same as in the case of symmetrical dynamics (by definition), but the quality of the segmentation is better. This difference comes from the gradient image quality which is not so good. In such case, the most precise the markers, the better the segmentation.

To solve this problem, we propose to use the information contained in the merging tree of extrema which is built when symmetrical dynamics is computed. Indeed, we have seen that a branch of the tree linking two nodes M and M' and created at level h corresponds to an inclusion relationship between the regions marked by M and M'. So, if 30 is the current level threshold, and if M is a selected extremum (its symmetrical dynamics is higher than 30), the marker we use is not M but the set composed by M and all the ascendants of M (see figure 12). This method allows to extract more precise markers without having to compute the contrast ASF. The associated segmentation is similar to the segmentation derived from the contrast ASF extrema (see figure 11).

Original image | Extrema of dynamics higher than 30 | Gradient image | Mosaic image (90 regions)

Figure 9: Segmentation of the most contrasted regions using dynamics

Original image | Extrema of sym. dynamics higher than 30 | Gradient image | Mosaic image (85 regions)

Figure 10: Segmentation of the most contrasted regions using symmetrical dynamics

contrast ASF of size 30 | Extended extrema | Gradient image | Mosaic image (85 regions)

Figure 11: Segmentation based on the computation of the contrast ASF

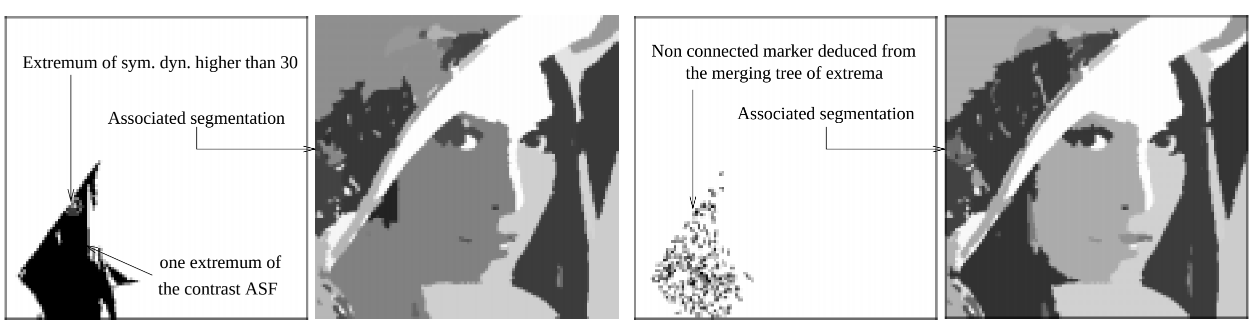

Figure 12: Extraction of non-connected markers using the merging tree of extrema

5 CONCLUSION

In this paper, we have introduced a symmetrical equivalent of dynamics based on a family of morphological alternating sequential filters. One characteristic of this transformation is that minima and maxima are studied simultaneously. The symmetrical dynamics we introduced is to the standard dynamics what alternating sequential filters are to open-close ot close-open operations.

We have also proposed an algorithm based on the construction of a merging tree of extrema. This tree essentially synthesizes the information contained in the successive images obtained by alternating sequential filtering and is therefore useful in segmentation applications.

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